





Woodland Academy Trust
Year 5 and Year 6 Calculation Document

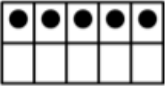




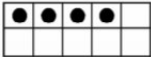






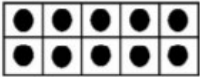
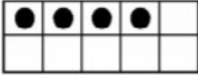





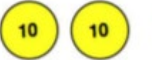







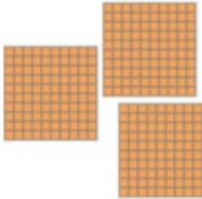


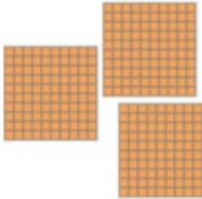


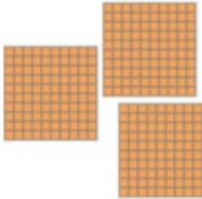


Progression in the use of manipulatives to support learning (How we support children's concrete understanding of maths)

Foundation	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Real-life objects	Real-life objects	Mini-whiteboards	Mini-whiteboards	Mini-whiteboards	Mini-whiteboards	Mini-whiteboards
0 – 9 digit cards	0 – 9 digit cards	Place value cards			Protractors	Protractors
Number track/line to 20	Number line to 20 and 50	Number line to 100	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick
		Transparent rulers	Transparent rulers	Transparent rulers	Transparent rulers	Transparent rulers
Tens frame	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square
Building blocks	Place value charts – Tens and ones	Place value charts – Ones to hundreds	Place value charts – Ones to Thousands	Place value charts – Ones to Ten thousands	Place value charts to a million and three decimal places	Place value charts to 10 million and three decimal places
Containers that are different shapes and sizes	Containers that are different shapes and sizes	Fraction bars, walls, circles (centralised storage)				
Numicon shapes	Numicon shapes/ Dienes	Dienes	Dienes	Dienes	Dienes	Dienes
Sorting hoops	Sorting hoops	Sorting hoops	Place value counters	Place value counters	Place value counters	Place value counters
Big Dice	Place value arrow cards – tens and ones	Place value arrow cards – tens and ones	Place value arrow cards – H, T, O	Place value arrow cards – H, T, O	Place value arrow cards	Place value arrow cards
Part-part-whole mat	Part-part-whole mat	Part-part-whole mat	Part-part-whole model	Part-part-whole model	Part-part-whole model	Part-part-whole model
Transparent counters	Transparent counters	Transparent counters	Transparent counters	Transparent counters	Transparent counters	Transparent counters
Bar model with real-life objects	Bar model pictorial objects/ representative objects e.g. counters	Bar model with counters /Dienes progressing to numbers	Plastic mirrors	Plastic mirrors	Plastic mirrors	Plastic mirrors
Bead strings – ten	Bead strings – twenty/fifty	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred
Dice	Dice	Dice	Dice	Dice	Dice	Dice
Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods
Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters
Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount
Maths balances			Weighing scales			
Solid geometric shapes (centralised storage)						
Coins and notes (centralised storage)						
Clock (geared) (centralised storage)						

Maths Working Wall (How we use displays to support children's understanding of mathematical concepts)		
Build it	Where possible use a real-life/concrete representation of the concept, which children can see, touch and feel.	
Draw it	Show a pictorial representation of the concept.	
Solve it	Show a worked example with success criteria to represent the mathematical concept.	$6 \times 2 = 12$ $2 \times 6 = 12$ $12 \div 2 = 6$ $12 \div 6 = 2$ Factors of 12 are: 1, 2, 3, 4, 6 and 12
Practise it	Encourage children to practice the concept. Interactive opportunity – ask children to respond to questions, encourage them to add what they know, leave homework for children to take to master the concept.	$1 \times 2 = 2$ $2 \times 2 = 4$ $3 \times 2 = 6$ etc.
Challenge it	Set a open-ended challenge to be solved. Interactive opportunity – leave real-life objects or manipulatives for children to use to help solve the challenge.	How many different ways can 12 eggs be arranged into arrays? What if you try 24 eggs?
Say it	Display and refer to the vocabulary related to the concept.	Multiply, multiplication , repeated addition, array, divide, group, multiples, factors

	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	<p>Combining two parts to make a whole: part whole model.</p> <p>Starting at the bigger number and counting on- using cubes.</p> <p>Regrouping to make 10 using ten frame.</p>	<p>Adding three single digits.</p> <p>Use of base 10 to combine two numbers.</p>	<p>Column method- regrouping.</p> <p>Using place value counters (up to 3 digits).</p>	<p>Column method- regrouping.</p> <p>(up to 4 digits)</p>	<p>Column method- regrouping.</p> <p>Use of place value counters for adding decimals.</p>	<p>Column method- regrouping.</p> <p>Abstract methods.</p> <p>Place value counters to be used for adding decimal numbers.</p>
Subtraction	<p>Taking away ones</p> <p>Counting back</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10 using the ten frame</p>	<p>Counting back</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10</p> <p>Use of base 10</p>	<p>Column method with regrouping.</p> <p>(up to 3 digits using place value counters)</p>	<p>Column method with regrouping.</p> <p>(up to 4 digits)</p>	<p>Column method with regrouping.</p> <p>Abstract for whole numbers.</p> <p>Start with place value counters for decimals- with the same amount of decimal places.</p>	<p>Column method with regrouping.</p> <p>Abstract methods.</p> <p>Place value counters for decimals- with different amounts of decimal places.</p>
Multiplication	<p>Recognising and making equal groups.</p> <p>Doubling</p> <p>Counting in multiples</p> <p>Use cubes, Numicon and other objects in the classroom</p>	<p>Arrays- showing commutative multiplication</p>	<p>Arrays</p> <p>$2d \times 1d$ using base 10</p>	<p>Column multiplication- introduced with place value counters.</p> <p>(2 and 3 digit multiplied by 1 digit)</p>	<p>Column multiplication</p> <p>Abstract only but might need a repeat of year 4 first (up to 4 digit numbers multiplied by 1 or 2 digits)</p>	<p>Column multiplication</p> <p>Abstract methods (multi-digit up to 4 digits by a 2 digit number)</p>
Division	<p>Sharing objects into groups</p> <p>Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups?</p> <p>Use cubes and draw round 3 cubes at a time.</p>	<p>Division as grouping</p> <p>Division within arrays- linking to multiplication</p> <p>Repeated subtraction</p>	<p>Division with a remainder- using lollipop sticks, times tables facts and repeated subtraction.</p> <p>$2d$ divided by $1d$ using base 10 or place value counters</p>	<p>Division with a remainder</p> <p>Short division (up to 3 digits by 1 digit- concrete and pictorial)</p>	<p>Short division</p> <p>(up to 4 digits by a 1 digit number including remainders)</p>	<p>Short division</p> <p>Long division with place value counters (up to 4 digits by a 2 digit number)</p> <p>Children should exchange into the tenths and hundredths column too</p>

Progression in the teaching of place value

Foundation	Year 1	Year 2	Year 3 onwards						
Understanding ten	Understanding numbers up to 20	Understanding numbers up to one hundred	Understanding numbers up to one thousand						
<p>A TENS FRAME is a simple maths tool that helps children:</p> <ul style="list-style-type: none"> • Keep track of counting • See number relationships • Learn addition to 10 • Understand place value <p>Use tens frames flash cards daily to ensure children recognise amounts.</p> <p>Use empty tens frames to fill with counters to enable children to understand number relationships.</p> <p>Either fill the tens frame in pairs or in rows. In rows shows 5 as a benchmark. Children can easily see more than 5 or less.</p> <div style="text-align: center;">  </div> <p>Setting the counters in pairs, naturally allows the children to see addition concepts.</p> <p>Include other visual images such as dice, cards, dominoes etc.</p> <div style="display: flex; justify-content: space-around; align-items: center;">    </div>	<p>'Ten' is the building block of our Base 10 numeration system. Young children can usually 'read' two-digit numbers long before they understand the effect the placement of each digit has on its numerical value. A child might be able to correctly read 62 as sixty two and 26 as twenty-six, and even know which number is larger, without understanding why the numbers are of differing values.</p> <p>Ten-frames can provide a first step into understanding two-digit numbers simply by the introduction of a second frame. Placing the second frame to the right of the first frame, and later introducing numeral cards, will further assist the development of place value understanding.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>	<p>Continue developing place value through the use of tens frames.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <div style="display: flex; justify-content: space-around; align-items: center;">  </div> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>	<p>Continue developing place value through the use of manipulatives including recognising 416 as 41 tens and 6 ones which is equivalent to 416 ones which is equivalent to four hundreds and one ten and six ones</p> <div style="text-align: center;">  </div> <div style="display: flex; justify-content: center; align-items: center;">    </div> <p>Use Dienes blocks and place value charts</p> <div style="text-align: center;"> <table border="1" style="margin: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Hundreds</th> <th style="padding: 5px;">Tens</th> <th style="padding: 5px;">Ones</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; vertical-align: middle;">  </td> <td style="text-align: center; vertical-align: middle;">  </td> <td style="text-align: center; vertical-align: middle;">  </td> </tr> </tbody> </table> </div>	Hundreds	Tens	Ones			
Hundreds	Tens	Ones							
									

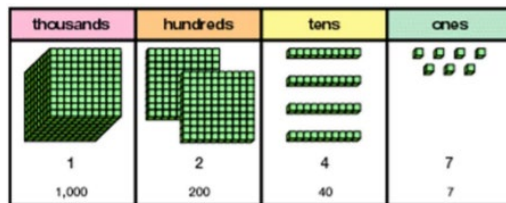
Progression in the teaching of place value

Year 4

Understanding numbers up to ten thousand

Continue developing place value through the use of manipulatives.

- Place value arrow cards
- Place value counters
- Dienes blocks
- Place value charts



Continue developing place value through the use of manipulatives including recognising the number above as one thousand plus two hundred plus four tens plus seven ones is equivalent to twelve hundred plus 47 ones etc. The children must also be able to identify that this number is also 12,470 tenths

Year 5

Understanding numbers up to one million including decimals

Continue developing place value through the use of manipulatives.

- Place value arrow cards
- Place value counters (including decimal counters)
- Dienes blocks
- Place value charts



They need to understand that there are no ten thousands in this number. The value of the digit 9 is nine thousand but there are three hundred and nine thousands in this number.

They need to be able to recognise the value of the digit and the number and know that these are different.

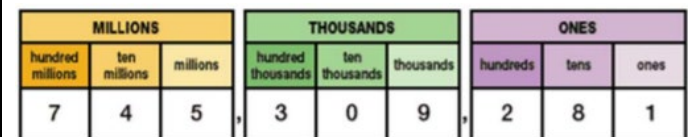
They also need to know how many tenths and hundredths are in this number 3092810 tenths and 30928100 hundredths in this number.

Year 6

Understanding numbers beyond one million including decimals

Continue developing place value through the use of manipulatives.



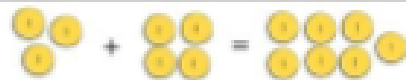


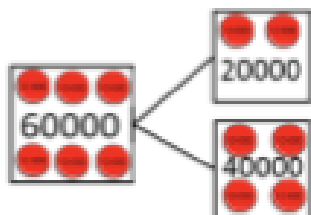
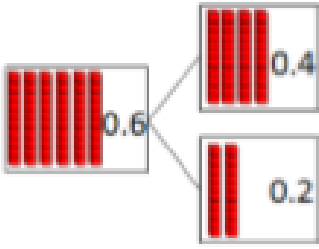
- Place value arrow cards
- Place value counters (including decimal counters)
- Dienes blocks
- Place value charts



They need to understand that there are no ten thousands in this number. The value of the digit 9 is nine thousand but there are 745309 thousands in this number.

They need to be able to recognise the value of the digit and the number and know that these are different. They also need to know how many tenths, hundredths and thousandths there are in this number 7453092810 tenths and 74530928100 hundredths and 745309281000 thousandths in this number.

Y5 and Y6 Addition & Subtraction

Strategies & Guidance	CPA																		
<p>Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000</p> <p><i>Skip counting forwards and backwards in steps of powers of 10 (i.e. 10, 100, 1000, 10 000 and 100 000) should be incorporated into transition activities and practised regularly.</i></p> <p><i>In Year 5 pupils work with numbers up to 1 000 000 as well as tenths, hundredths and thousandths.</i></p> <p><i>In Year 6 pupils work with numbers up to 10 000 000.</i></p>	<p>Support with place value counters on a place value chart, repeatedly adding the same counter and regrouping as needed.</p> <table border="1" style="width: 100%; text-align: center; border-collapse: collapse;"> <tr> <td style="width: 12.5%;">Hundred Thousands</td> <td style="width: 12.5%;">Ten Thousands</td> <td style="width: 12.5%;">Thousands</td> <td style="width: 12.5%;">Hundreds</td> <td style="width: 12.5%;">Tens</td> <td style="width: 12.5%;">Units</td> <td style="width: 12.5%;">Tenths</td> <td style="width: 12.5%;">Hundredths</td> <td style="width: 12.5%;">Thousandths</td> </tr> <tr> <td style="height: 40px;"></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table> <p>Counting sticks and number lines:</p>   <p>Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes. e.g. $9900 + 100 = 10\ 000$ or $99\ 000 + 1000 = 100\ 000$</p>	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths									
Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths											
<p>Using known facts and understanding of place value to derive</p> <p><i>Using the following language makes the logic explicit: I know three ones plus four ones is equal to seven ones. Therefore, three ten thousands plus four ten thousands is equal to seven ten thousands.</i></p> <p><i>In Year 5 extend to multiples of 10 000 and 100 000 as well as tenths, hundredths and thousandths.</i></p> <p><i>In Year 6 extend to multiples of one million.</i></p> <p><i>These derived facts should be used to estimate and check answers to calculations.</i></p>	<div style="display: flex; justify-content: space-between;"> <div style="text-align: center;">  <p>$3 + 4 = 7$</p> </div> <div style="text-align: center;">  <p>$30\ 000 + 40\ 000 = 70\ 000$</p> </div> <div style="text-align: center;">  <p>$300\ 000 + 400\ 000 = 700\ 000$</p> </div> </div> <div style="margin-top: 20px;"> <p>$20\ 000 + 40\ 000 = 60\ 000$</p> <p>$40\ 000 + 20\ 000 = 60\ 000$</p> <p>$60\ 000 - 40\ 000 = 20\ 000$</p> <p>$60\ 000 - 20\ 000 = 40\ 000$</p> </div> <div style="margin-top: 20px;">  </div> <div style="margin-top: 20px;">  <div style="display: flex; justify-content: space-between; margin-left: 100px;"> <div style="text-align: center;"> <p>$0.6 = 0.2 + 0.4$</p> <p>$0.6 = 0.4 + 0.2$</p> <p>$0.2 = 0.6 - 0.4$</p> <p>$0.4 = 0.6 - 0.2$</p> </div> </div> </div>																		

Strategies & Guidance**CPA****Partitioning one number and applying known facts to add.**

Pupils can use this strategy mentally or with jottings as needed.

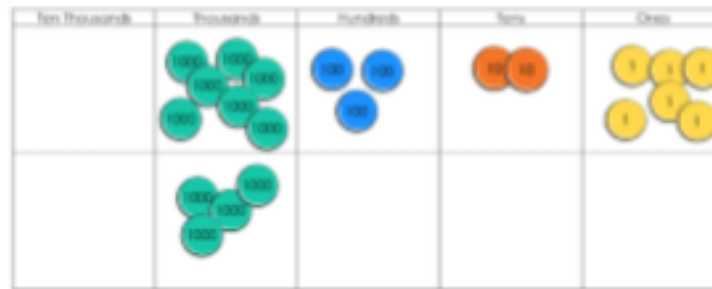
Pupils should be aware of the range of choices available when deciding how to partition the number that is to be added.

They should be encouraged to count on from the number of greater value as this will be more efficient. However, they should have an understanding of the commutative law of addition, that the parts can be added in any order.

Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.

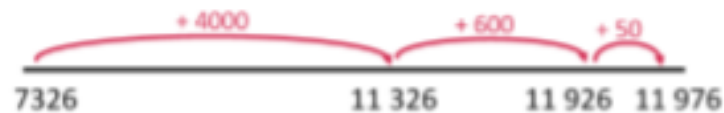
Partitioning into place value amounts (canonical partitioning):

$$4650 + 7326 = 7326 + 4000 + 600 + 50$$



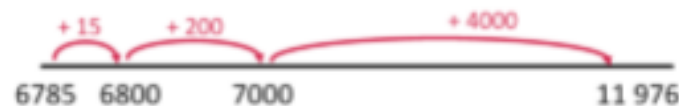
With place value counters, represent the larger number and then add each place value part of the other number. The image above shows the thousands being added.

Represent pictorially with an empty numberline:

**Partitioning in different ways (non-canonical partitioning):**

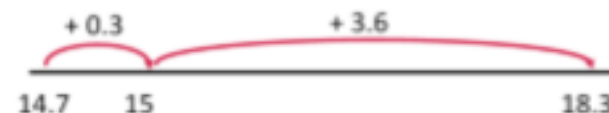
Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count on to a multiple of 10.

$$6785 + 2325 = 6785 + 15 + 200 + 2110$$



The strategy can be used with decimal numbers, Make one:

$$14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3$$



Strategies & Guidance**CPA****Subtraction by partitioning and applying known facts.**

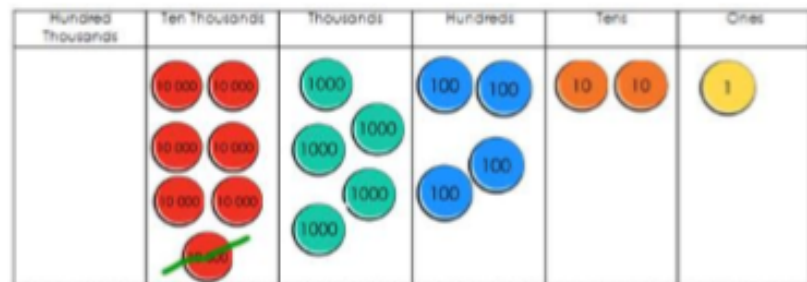
Pupils can use this strategy mentally or with jottings as needed.

Pupils should be aware of the range of choices available when deciding how to partition the number that is to be subtracted.

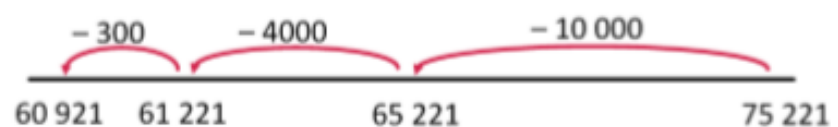
Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.

Partitioning into place value amounts (canonical partitioning):

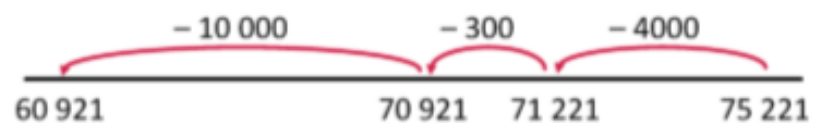
$$75\ 221 - 14\ 300 = 75\ 221 - 10\ 000 - 4000 - 300$$



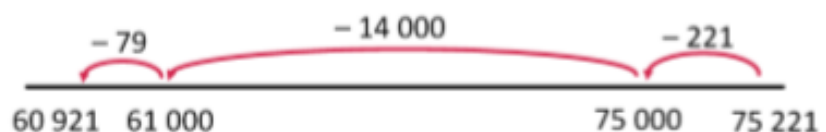
Represent pictorially with a number line, starting on the right and having the arrows jump to the left:

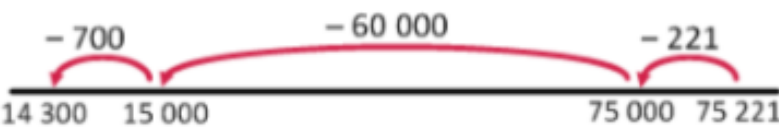
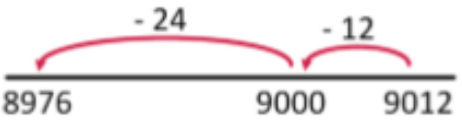
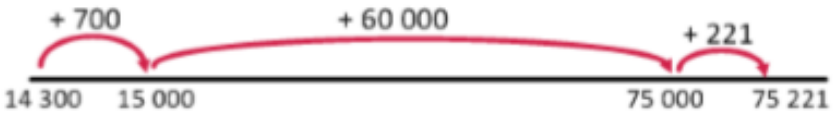
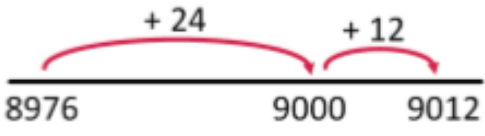


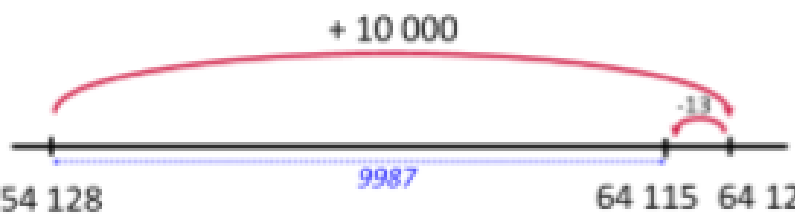
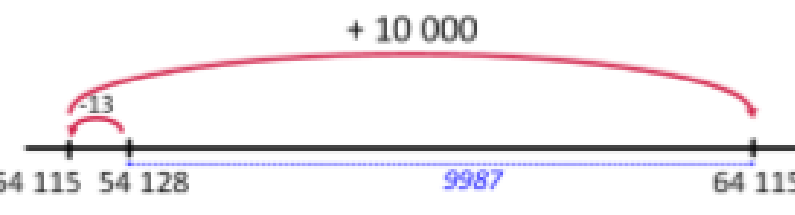
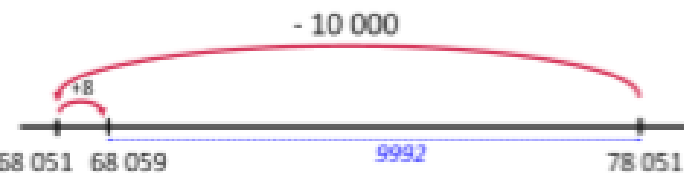
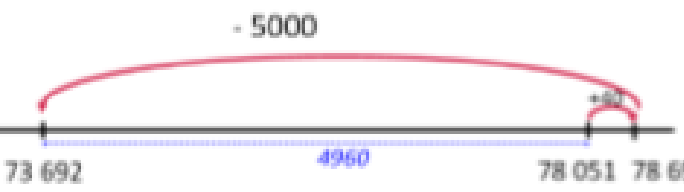
Develop understanding that the parts can be subtracted in any order and the result will be the same:

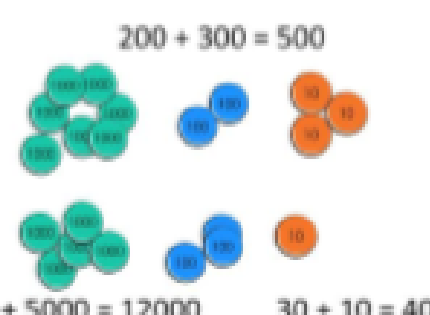
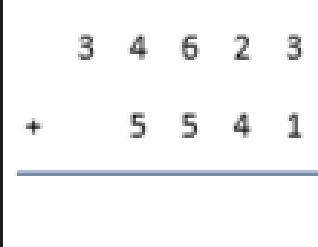
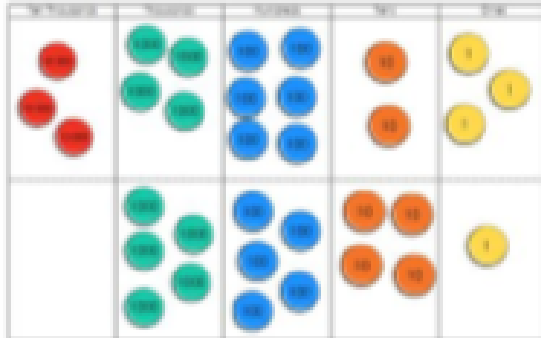
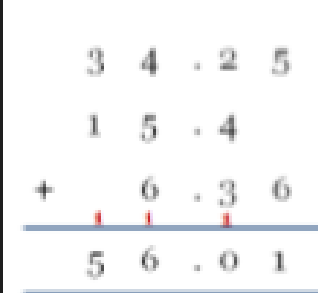
**Partitioning in different ways (non-canonical partitioning):**

Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count back to a multiple of 10.



Strategies & Guidance	CPA
<p>Calculate difference by “counting back”</p> <p><i>It is interesting to note that finding the difference is reversible. For example, the difference between 5 and 2 is the same as the difference between 2 and 5. This is not the case for other subtraction concepts.</i></p>	<p>$75\,221 - 14\,300$</p> <p>Place the numbers either end of a numberline and work out the difference between them. Select efficient jumps.</p>  <p>Finding the difference is efficient when the numbers are close to each other:</p> <p>$9012 - 8976$</p> 
<p>Calculate difference by “counting on”</p> <p><i>Addition strategies can be used to find difference.</i></p>	<p>$75\,221 - 14\,300$</p>  <p>Finding the difference is efficient when the numbers are close to each other</p> <p>$9012 - 8976$</p> 





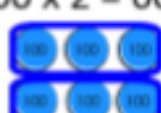



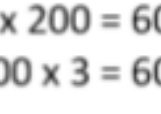
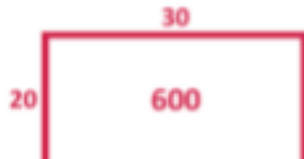
Strategies & Guidance	CPA
<p>Round and adjust</p> <p>Addition and subtraction using compensation</p> <p><i>Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.</i></p> <p><i>It is very easy to be confused about how to adjust and so visual representations and logical reasoning are essential to success with this strategy.</i></p>	<p>Addition</p>  <p>$54\ 128 + 9987 = 54\ 128 + 10\ 000 - 13 = 64\ 128 - 13$</p> <p>Pupils should realise that they can adjust first:</p>  <p>$54\ 128 + 9987 = 54\ 128 - 13 + 10\ 000 = 54\ 115 + 10\ 000$</p> <p>Subtraction</p>  <p>$78\ 051 - 9992 = 78\ 051 - 10\ 000 + 8 = 68\ 051 + 8$</p> <p>Pupils should realise that they can adjust first:</p>  <p>$78\ 051 - 4960 = 78\ 051 + 641 - 5000 = 78\ 692 - 5000$</p>
<p>Near doubles</p> <p><i>Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten as well as decimal numbers. These facts can be adjusted to calculate near doubles.</i></p>	<p>$160 + 170 = \text{double } 150 + 10 + 20$</p> <p>$160 + 170 = \text{double } 160 + 10 \quad \text{or} \quad 160 + 170 = \text{double } 170 - 10$</p> <p>$2.5 + 2.6 = \text{double } 2.5 + 0.1$</p>

Strategies & Guidance	CPA
<p>Partition both numbers and combine the parts</p> <p><i>Pupils should be secure with this method for numbers up to 10 000, using place value counters or Dienes to show conceptual understanding.</i></p> <p><i>If multiple regroupings are required, then pupils should consider using the column method.</i></p>	<p>$7230 + 5310 = 12\ 000 + 500 + 40$</p> <p>$200 + 300 = 500$</p>  <p>$7000 + 5000 = 12000$ $30 + 10 = 40$</p> <p>Pupils should be aware that the parts can be added in any order.</p>
<p>Written column methods for addition</p> <p><i>In Year 5, pupils are expected to be able to use formal written methods to add whole numbers with more than four digits as well as working with numbers with up to three decimal places.</i></p> <p><i>Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective.</i></p> <p><i>Continue to use concrete manipulatives alongside the formal method.</i></p> <p><i>When adding decimal numbers with a different number of decimal places, in order to avoid calculation errors, pupils should be encouraged to insert zeros so that there is a digit in every row. This is not necessary for calculation and these zeros are not place holders as the value of the other digits is not changed by it being placed.</i></p> <p><i>Exemplification of this method and the language to use are best understood through viewing the tutorial videos found here on the toolkit.</i></p>	<p>For this method start with the digit of least value because if regrouping happens it will affect the digits of greater value.</p>  <p>Combine the counters in each column and regroup as needed:</p>  <p>Decimal numbers:</p> 

Strategies & Guidance	CPA
<p>Written column methods for subtraction</p> <p><i>In Year 5, pupils are expected to be able to use formal written methods to subtract whole numbers with more than four digits as well as working with numbers with up to three decimal places.</i></p> <p><i>Pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping.</i></p> <p><i>In Year 3 and 4 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.</i></p> <p><i>Pupils should think about if this is the most efficient method, considering whether mental strategies (such as counting on, using known number facts, compensation etc.) may be likelier to produce an accurate solution.</i></p> <p><i>Exemplification of this method and the language to use are best understood through viewing the tutorial videos found here on the toolkit.</i></p>	<div style="text-align: center;">CPA</div> <div style="display: flex; justify-content: space-between; align-items: flex-start;"> <div style="text-align: right;"> $\begin{array}{r} 41362 \\ - 32243 \\ \hline \end{array}$ $\begin{array}{r} 3\cancel{4}13\cancel{5}12 \\ - 32243 \\ \hline 9119 \end{array}$ </div> <div style="text-align: left;"> </div> </div> <p>The term regrouping should be the language used. You can use the terms 'exchange' with subtraction but it needs careful consideration.</p> <p>You can regroup 62 as 50 and 12 (5 tens and 12 ones) instead of 60 and 2 (6 tens and 12 ones).</p> <p>Or you can 'exchange' one of the tens for 10 ones resulting in 5 tens and 12 ones.</p> <p>If you have exchanged, then the number has been regrouped.</p>

Y5 and Y6 Multiplication


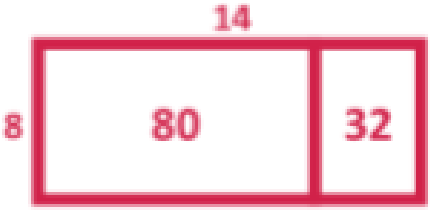




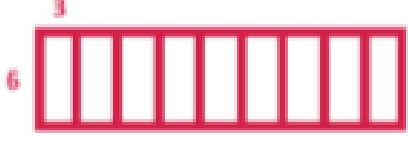
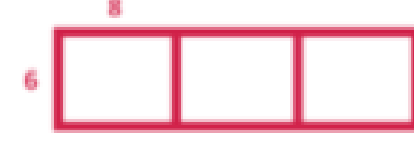
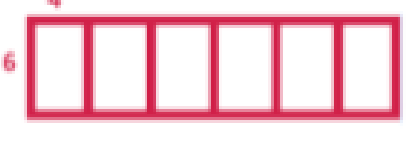
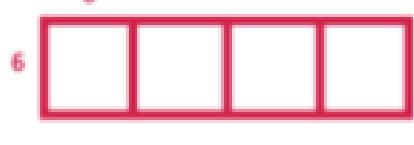
Strategies & Guidance	CPA																																							
<p>Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000</p> <p><i>Avoid saying that you “add a zero” when multiplying by ten and instead use the language of place holder.</i></p> <p><i>Use place value counters and charts to visualise and then notice what happens to the digits.</i></p>	<p>When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc.</p> <p>When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.</p> <p>$102.14 \times 10 = 1021.4$</p> <table border="1" style="width: 100%; text-align: center; border-collapse: collapse;"> <thead> <tr> <th>Thousands</th> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> <th>•</th> <th>Tenths</th> <th>Hundredths</th> </tr> </thead> <tbody> <tr> <td></td> <td>100 100 100</td> <td></td> <td>1 1</td> <td>•</td> <td>0.01</td> <td>0.1 0.1 0.1 0.1</td> </tr> <tr> <td>1000 1000 1000</td> <td></td> <td>10 10</td> <td>1</td> <td>•</td> <td>0.01 0.01 0.01 0.01</td> <td></td> </tr> </tbody> </table> <p>When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.</p> <p>When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller.</p> <p>E.g. $210 \div 10 = 21$</p> <p>$210.3 \div 10 = 21.03$</p> <table border="1" style="width: 100%; text-align: center; border-collapse: collapse;"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> <th>•</th> <th>Tenths</th> <th>Hundredths</th> </tr> </thead> <tbody> <tr> <td>100 100</td> <td>10</td> <td></td> <td>•</td> <td>0.1 0.1 0.1 0.1</td> <td></td> </tr> <tr> <td></td> <td>10 10</td> <td>1</td> <td>•</td> <td></td> <td>0.01 0.01 0.01 0.01</td> </tr> </tbody> </table>	Thousands	Hundreds	Tens	Ones	•	Tenths	Hundredths		100 100 100		1 1	•	0.01	0.1 0.1 0.1 0.1	1000 1000 1000		10 10	1	•	0.01 0.01 0.01 0.01		Hundreds	Tens	Ones	•	Tenths	Hundredths	100 100	10		•	0.1 0.1 0.1 0.1			10 10	1	•		0.01 0.01 0.01 0.01
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Strategies & Guidance	CPA
<p>Using known facts and place value to derive multiplication facts</p> <p><i>Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing for larger 'fact families' to be derived from a single known number fact.</i></p> <p><i>Knowledge of commutativity is further extended and applied to find a range of related facts.</i></p> <p><i>Pupils should work with decimals with up to two decimal places.</i></p> <p><i>These derived facts should be used to estimate and check answers to calculations.</i></p>	<div style="text-align: center;">  </div> <p>$2 \times 3 = 6$ $3 \times 2 = 6$</p> <p>$2 \times 30 = 60$ $30 \times 2 = 60$</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <p>$2 \times 3 \times 10 = 60$</p> <p>$3 \times 20 = 60$ $20 \times 3 = 60$</p> <p>$2 \times 300 = 600$ $300 \times 2 = 600$</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <p>$2 \times 30 \times 10 = 600$ $20 \times 3 \times 10 = 600$</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <p>$3 \times 200 = 600$ $200 \times 3 = 600$</p> <p>$20 \times 30 = 600$ $30 \times 20 = 600$</p>

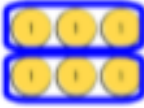

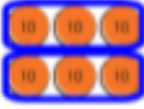

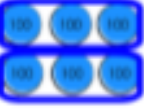





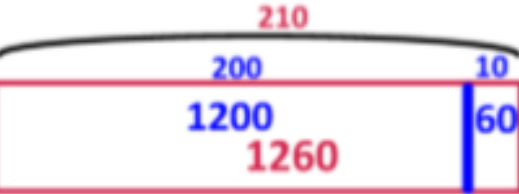
These are the multiplication facts pupils should be able to derive from a known fact



2 100 000	700 000 x 3	70 000 x 30	7000 x 300	700 x 3000	70 x 30 000	7 x 300 000
210 000	70 000 x 3	7000 x 30	700 x 300	70 x 3000	7 x 30 000	
21 000	7000 x 3	700 x 30	70 x 300	7 x 3000		
2100	700 x 3	70 x 30	7 x 300			
210	70 x 3	7 x 30				
21 = 7 x 3						
2.1	0.7 x 3	7 x 0.3				
0.21	0.07 x 3	0.7 x 0.3	7 x 0.03			
0.021	0.007 x 3	0.07 x 0.3	0.7 x 0.03	7 x 0.003		

Strategies & Guidance	CPA
<p>Doubling and halving</p> <p><i>Pupils should experience doubling and halving larger and smaller numbers as they expand their understanding of the number system.</i></p> <p><i>Doubling and halving can then be used in larger calculations.</i></p>	<div data-bbox="715 170 1305 398" style="text-align: center;"> </div> <p>Multiply by 4 by doubling and doubling again e.g. $16 \times 4 = 32 \times 2 = 64$</p> <p>Divide by 4 by halving and halving again e.g. $104 \div 4 = 52 \div 2 = 26$</p> <div data-bbox="616 721 1388 976" style="text-align: center;"> </div> <p>Multiply by 8 by doubling three times e.g. $12 \times 8 = 24 \times 4 = 48 \times 2 = 96$</p> <p>Divide by 8 by halving three times e.g. $104 \div 8 = 52 \div 4 = 26 \div 2 = 13$</p> <div data-bbox="561 1308 1455 1491" style="text-align: center;"> </div> <p>Multiply by 5 by multiplying by 10 then halving, e.g. $18 \times 5 = 180 \div 2 = 90$.</p> <p>Divide by 5 by dividing by 10 and doubling, e.g. $460 \div 5 = \text{double } 46 = 92$</p>

Strategies & Guidance	CPA
<p>Multiply by partitioning one number and multiplying each part</p> <p>Distributive law</p> <p>$a \times (b + c) = a \times b + a \times c$</p> <p>Build on pupils' understanding of arrays of counters to represent multiplication to see that area models can be a useful representation:</p>	<p>$8 \times 14 = 8 \times 10 + 8 \times 4$</p>  <p>80 32</p> <p>Cuisenaire rods to build arrays</p>  <p>8 14 80 32</p> <p>Represent with area model</p>  <p>0 80 100 112</p> <p>Jottings on a number line</p> <p>Bead string where each bead has a value of 8:</p>  <p>80 32</p>
<p>Using knowledge of factors</p> <p><i>In Year 5 pupils are expected to be able to identify factor pairs and this knowledge can be used to calculate.</i></p> <p><i>Pupils will be using the commutative and associative laws of multiplication.</i></p> <p>Commutative law</p> <p>$a \times b = b \times a$</p> <p>Associative law</p> <p>$a \times b \times c = (a \times b) \times c$</p> <p>$= a \times (b \times c)$</p> <p><i>They should explore and compare the different options and choose the most efficient order to complete calculations.</i></p>	<p>Calculate 6×24 by using factor pairs of 24</p> <p>Two and twelve are factors of 24:</p> <p>$6 \times 2 \times 12$ $6 \times 12 \times 2$</p>   <p>Three and eight are factors of 24:</p> <p>$6 \times 3 \times 8$ $6 \times 8 \times 3$</p>   <p>Four and six are factors of 24:</p> <p>$6 \times 4 \times 6$ $6 \times 6 \times 4$</p>  

Y5 and Y6 Division

Strategies & Guidance	CPA
<p>Deriving facts from known facts</p> <p><i>Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.</i></p> <p><i>Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.</i></p>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> $6 \div 2 = 3$  </div> <div style="text-align: center;"> $6 \div 3 = 2$  </div> </div> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="text-align: center;"> $60 \div 2 = 30$ $60 \div 30 = 2$ </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> <div style="text-align: center;"> $60 \div 3 = 20$ $60 \div 20 = 3$ </div> </div> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="text-align: center;"> $600 \div 2 = 300$ $600 \div 300 = 2$ </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> <div style="text-align: center;"> $600 \div 3 = 200$ $600 \div 200 = 3$ </div> </div>
<p>Using knowledge of multiples to divide</p> <p><i>Using an area model to partition the whole into multiples of the divisor (the number you are dividing by).</i></p>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>80 32</p> </div> <div style="text-align: center;"> $112 \div 8 = 80 \div 8 + 32 \div 8$ </div> </div> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 20px;"> <div style="text-align: center;"> $1260 \div 6 = 1200 \div 6 + 60 \div 6$ </div> </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;"> <div style="text-align: center;"> <p>How many equal parts?</p>  <p>6 1260</p> </div> </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;"> <div style="text-align: center;">  </div> </div>

Strategies & Guidance	CPA
<p>Using knowledge of factors to divide</p> <p><i>Pupils explore this strategy when using repeated halving.</i></p> <p><i>$2 \times 2 = 4$ and so if you divide by 4 the same result can be achieved by dividing by two and then by two again.</i></p>	<div style="text-align: center;"> <p>24</p>  <p>144 ÷ 24</p> </div> <div style="text-align: center; margin-top: 20px;"> <p>12 12</p>  <p>144 ÷ 2 ÷ 12</p> </div> <p>I know 2 and 12 are a factor pair of 24 and so I can divide by 2 and then by 12.</p>

Strategies & Guidance	CPA																												
<p>Short division</p> <p>Dividing a 4-digit numbers by 1-digit numbers</p> <p><i>The thought process of the traditional algorithm is as follows:</i></p> <p><i>How many 4s in 8? 2</i> <i>How many 4s in 5? 1 with 1 remaining so regroup.</i> <i>How many 4s in 12? 3</i> <i>How many 4s in 8? 2</i></p> <p><i>Warning: If you simply apply place value knowledge to each step, the thinking goes wrong if you have to regroup.</i></p> <p><i>How many 4s in 8000? 2000</i> <i>How many 4s in 500? 100 with 1 remaining (illogical)</i> <i>The answer would be 125.</i></p> <p><i>Sharing the dividend builds conceptual understanding however doesn't scaffold the "thinking" of the algorithm.</i></p> <p><i>Using place value counters and finding groups of the divisor for each power of ten will build conceptual understanding of the short division algorithm.</i></p> <p><i>Area models are also useful representations, as seen with other strategies and exemplified for long division.</i></p> <p><i>Exemplification of this method and the language to use are best understood through viewing the tutorial videos found here on the toolkit.</i></p>	<p style="text-align: right;">CPA</p> <p>8528 ÷ 4</p> <div style="text-align: right; margin-left: 200px;"> $\begin{array}{r} 2132 \\ 4 \overline{) 8528} \end{array}$ </div> <p>Sharing</p> <table style="width: 100%; text-align: center; border-collapse: collapse;"> <thead> <tr> <th style="border-bottom: 1px solid black;">Thousands</th> <th style="border-bottom: 1px solid black;">Hundreds</th> <th style="border-bottom: 1px solid black;">Tens</th> <th style="border-bottom: 1px solid black;">Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>8 thousands shared into 4 equal groups 5 hundreds shared into 4 equal groups Regroup 1 hundred for 10 tens 12 tens shared into 4 equal groups 8 ones shared into 4 equal groups.</p> <p>Grouping</p> <table style="width: 100%; text-align: center; border-collapse: collapse;"> <thead> <tr> <th style="border-bottom: 1px solid black;">Thousands</th> <th style="border-bottom: 1px solid black;">Hundreds</th> <th style="border-bottom: 1px solid black;">Tens</th> <th style="border-bottom: 1px solid black;">Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>How many groups of 4 thousands in 8 thousands? How many groups of 4 hundreds in 5 hundreds? Regroup 1 hundred for 10 tens. How many groups of 4 tens in 12 tens? How many groups of 4 ones in 8 ones?</p>	Thousands	Hundreds	Tens	Ones																	Thousands	Hundreds	Tens	Ones				
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Strategies & Guidance	CPA
<p>Long division</p> <p>Dividing a 4-digit number by a 2-digit number</p> <p><i>Follow the language structures of the short division strategy. Instead of recording the regrouped amounts as small digits the numbers are written out below. This can be easier to work with when dividing by larger numbers.</i></p> <p><i>If dividing by a number outside of their known facts, pupils should start by recording some multiples of that number to scaffold.</i></p>	<div style="text-align: right; margin-bottom: 20px;"> $\begin{array}{r} 34 \\ 12 \overline{) 408} \\ \underline{36} \\ 48 \\ \underline{48} \\ 0 \end{array}$ </div> <p>$408 \div 12$</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> $30 \times 12 = 360$ </div> <div style="text-align: center;"> $4 \times 12 = 48$ </div> </div>