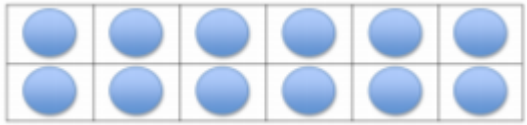
Woodland Academy Trust

Year 3 Calculation Document

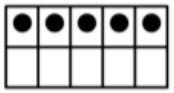
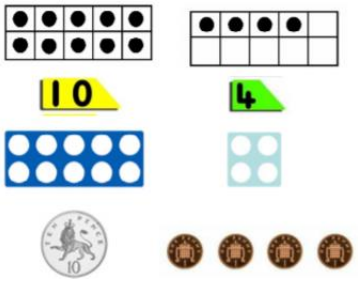
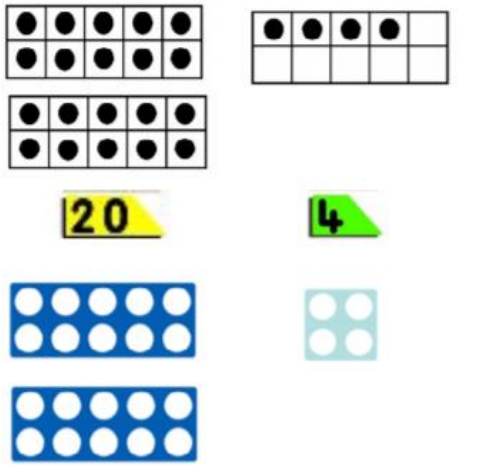
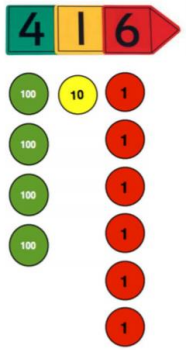
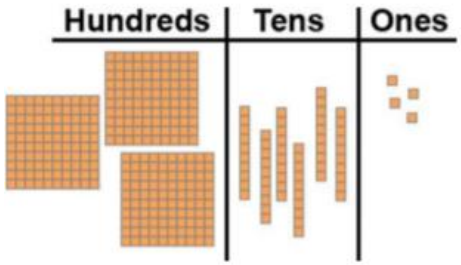
Progression in the use of manipulatives to support learning (How we support children's concrete understanding of maths)						
Foundation	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
Real-life objects	Real-life objects	Mini-whiteboards	Mini-whiteboards	Mini-whiteboards	Mini-whiteboards	Mini-whiteboards
0 – 9 digit cards	0 – 9 digit cards	Place value cards			Protractors	Protractors
Number track/line to 20	Number line to 20 and 50	Number line to 100	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick
		Transparent rulers	Transparent rulers	Transparent rulers	Transparent rulers	Transparent rulers
Tens frame	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square
Building blocks	Place value charts – Tens and ones	Place value charts – Ones to hundreds	Place value charts – Ones to Thousands	Place value charts – Ones to Ten thousands	Place value charts to a million and three decimal places	Place value charts to 10 million and three decimal places
Containers that are different shapes and sizes	Containers that are different shapes and sizes	Fraction bars, walls, circles (centralised storage)				
Numicon shapes	Numicon shapes/ Dienes	Dienes	Dienes	Dienes	Dienes	Dienes
Sorting hoops	Sorting hoops	Sorting hoops	Place value counters	Place value counters	Place value counters	Place value counters
Big Dice	Place value arrow cards – tens and ones	Place value arrow cards – tens and ones	Place value arrow cards – H, T, O	Place value arrow cards – H, T, O	Place value arrow cards	Place value arrow cards
Part-part-whole mat	Part-part-whole mat	Part-part-whole mat	Part-part-whole model	Part-part-whole model	Part-part-whole model	Part-part-whole model
Transparent counters	Transparent counters	Transparent counters	Transparent counters	Transparent counters	Transparent counters	Transparent counters
Bar model with real-life objects	Bar model pictorial objects/ representative objects e.g. counters	Bar model with counters /Dienes progressing to numbers	Plastic mirrors	Plastic mirrors	Plastic mirrors	Plastic mirrors
Bead strings – ten	Bead strings – twenty/fifty	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred
Dice	Dice	Dice	Dice	Dice	Dice	Dice
Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods
Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters
Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount
Maths balances			Weighing scales			
Solid geometric shapes (centralised storage)						
Coins and notes (centralised storage)						
Clock (geared) (centralised storage)						

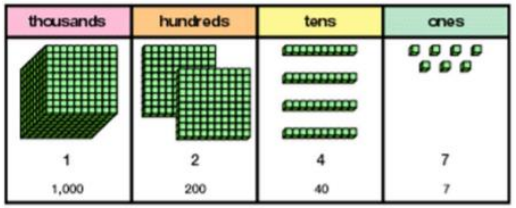
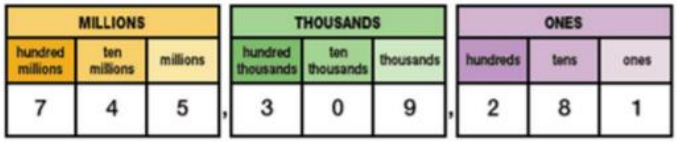
Maths Working Wall (How we use displays to support children's understanding of mathematical concepts)		
Build it	Use a real-life representation of the concept, which children can see, touch and feel.	
Draw it	Show a pictorial representation of the concept.	
Solve it	Show the mathematical representation of the concept	6 x 2 = 12 2 x 6 = 12 12 ÷ 2 = 6 12 ÷ 6 = 2 Factors of 12 are: 1, 2, 3, 4, 6 and 12
Practise it	Encourage children to practice the concept. Interactive opportunity – ask children to respond to questions, encourage them to add what they know, leave homework for children to take to master the concept.	1 x 2 = 2 2 x 2 = 4 3 x 2 = 6 etc.
Challenge it	Set a challenge to be solved. Interactive opportunity – leave real-life objects or manipulatives for children to use to help solve the challenge.	How many different ways can 12 eggs be arranged into arrays? What if you try 24 eggs?
Say it	Use vocabulary related to the concept	Multiply, multiplication , repeated addition, array, divide, group, multiples, factors

Classroom visual prompts (How we represent maths to the children pictorially)						
Foundation	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
Big focus 10	Big focus 20	Big focus 100				
Place Value Chart 10	Place Value Chart 20	Place Value Chart 100	Place Value Chart Th- tenths	Place Value Chart Tth- Hundredths	Place Value Chart M- Thousandths	Place Value Chart M- Thousandths
Numicon number line with Numicon shapes	Numicon number line with Numicon shapes	Fractions number line	Fractions number line	Fractions and decimals number line	Fractions, decimals and percentages number line	Fractions, decimals and percentages number line
Odd and even numbers	Odd and even numbers	Odd and even numbers	Factors and multiples	Factors and multiples	Factors, prime and composite numbers	Number properties
	Number bonds to 10 Number bonds to 20	Number bonds to 10 Multiples of 10 totalling 100	Number bonds to 10 Multiples of 10 totalling 100			
0 – 20 number line / track	0 -50 number line	0 – 100 number line	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
	100 square	100 square	100 square	100 square	100 square	100 square
Number names from 0 - 10	Number names of multiples of 10	Number names from 0 – 100	Number names from 0 - 1000	Number names to hundred thousands	Number names to one million	Number names to million
Real coins and Large coins	Real coins and Large coins	Real coins and Large coins	Real coins and Large coins	Real coins and Large coins	Real coins and Large coins	Real coins and Large coins
Counting in 1s and 2s	2, 5 and 10 multiplication tables	2, 4 and 8 multiplication tables	3, 6 and 12 multiplication tables	7, 9 and 11 multiplication tables All multiplication tables up to 12 x 12	All multiplication tables up to 12 x 12	All multiplication tables up to 12 x 12
Counting in 1s and 2s multiplication table patterns and divisibility rules and connections.	2, 5 and 10 multiplication table patterns and divisibility rules and connections. Display after introducing the times tables to the children.	2, 4 and 8 multiplication table patterns and divisibility rules and connections. Display after introducing the times tables to the children.	3, 6 and 12 multiplication table patterns and divisibility rules and connections. Display after introducing the times tables to the children.	All multiplication table patterns and divisibility rules Connections between 5/10, 3/6/12, 2/4/8 Also focus on 1, 7, 9 and 0 multiplication table.	All multiplication table patterns and divisibility rules Connections between 5/10, 3/6/12, 2/4/8 Also focus on 1, 7, 9 and 0 multiplication table. Square and cube numbers	All multiplication table patterns and divisibility rules Connections between 5/10, 3/6/12, 2/4/8 Also focus on 1, 7, 9 and 0 multiplication table. Square and cube numbers
			Roman numerals	Roman numerals	Roman numerals	Roman numerals
The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to
2D and 3D shapes	2D and 3D shapes	2D and 3D shapes	2D and 3D shapes	2D and 3D shapes	2D and 3D shapes	2D and 3D shapes

	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method- regrouping. Using place value counters (up to 3 digits).	Column method- regrouping. (up to 4 digits)	Column method- regrouping. Use of place value counters for adding decimals.	Column method- regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals- with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.
Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Arrays 2d × 1d using base 10	Column multiplication- introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first(up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too

Progression in the teaching of place value

Foundation	Year 1	Year 2	Year 3 onwards
Understanding ten	Understanding numbers up to 20	Understanding numbers up to one hundred	Understanding numbers up to one thousand
A TENS FRAME is a simple maths tool that helps children: • Keep track of counting • See number relationships • Learn addition to 10 • Understand place value Use tens frames flash cards daily to ensure children recognise amounts. Use empty tens frames to fill with counters to enable children to understand number relationships. Either fill the tens frame in pairs or in rows. In rows shows 5 as a benchmark. Children can easily see more than 5 or less.  Setting the counters in pairs, naturally allows the children to see addition concepts. Include other visual images such as dice, cards, dominoes etc. 	'Ten' is the building block of our Base 10 numeration system. Young children can usually 'read' two-digit numbers long before they understand the effect the placement of each digit has on its numerical value. A child might be able to correctly read 62 as sixty two and 26 as twenty-six, and even know which number is larger, without understanding why the numbers are of differing values. Ten-frames can provide a first step into understanding two-digit numbers simply by the introduction of a second frame. Placing the second frame to the right of the first frame, and later introducing numeral cards, will further assist the development of place value understanding. 	Continue developing place value through the use of tens frames. 	Continue developing place value through the use of manipulatives including recognising 416 as 41 tens and 6 ones which is equivalent to 416 ones which is equivalent to four hundreds and one ten and six ones  Use Dienes blocks and place value charts 

Progression in the teaching of place value		
Year 4	**Year 5**	**Year 6**
Understanding numbers up to ten thousand	**Understanding numbers up to one million including decimals**	**Understanding numbers beyond one million including decimals**
Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters • Dienes blocks • Place value charts  Continue developing place value through the use of manipulatives including recognising the number above as one thousand plus two hundred plus four tens plus seven ones is equivalent to twelve hundred plus 47 ones etc. The children must also be able to identify that this number is also 12,470 tenths	Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters (including decimal counters) • Dienes blocks • Place value charts  They need to understand that there are no ten thousands in this number. The value of the digit 9 is nine thousand but there are three hundred and nine thousands in this number. They need to be able to recognise the value of the digit and the number and know that these are different. They also need to know how many tenths and hundredths are in this number 3092810 tenths and 30928100 hundredths in this number.	Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters (including decimals counters) • Dienes blocks • Place value charts  They need to understand that there are no ten thousands in this number. The value of the digit 9 is nine thousand but there are 745309 thousands in this number. They need to be able to recognise the value of the digit and the number and know that these are different. They also need to know how many tenths, hundredths and thousandths there are in this number 7453092810 tenths and 74530928100 hundredths and 745309281000 thousandths in this number.

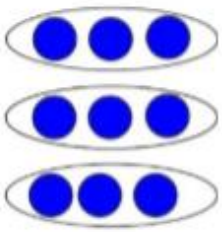
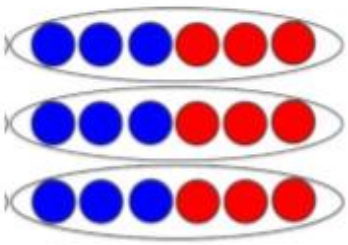
Y3 Addition & Subtraction

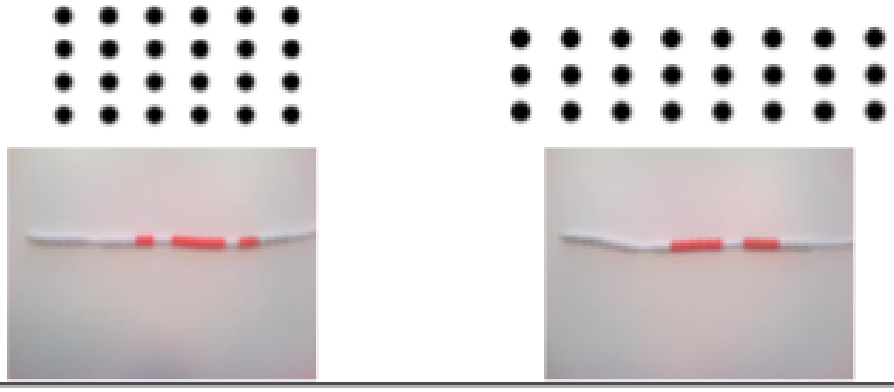
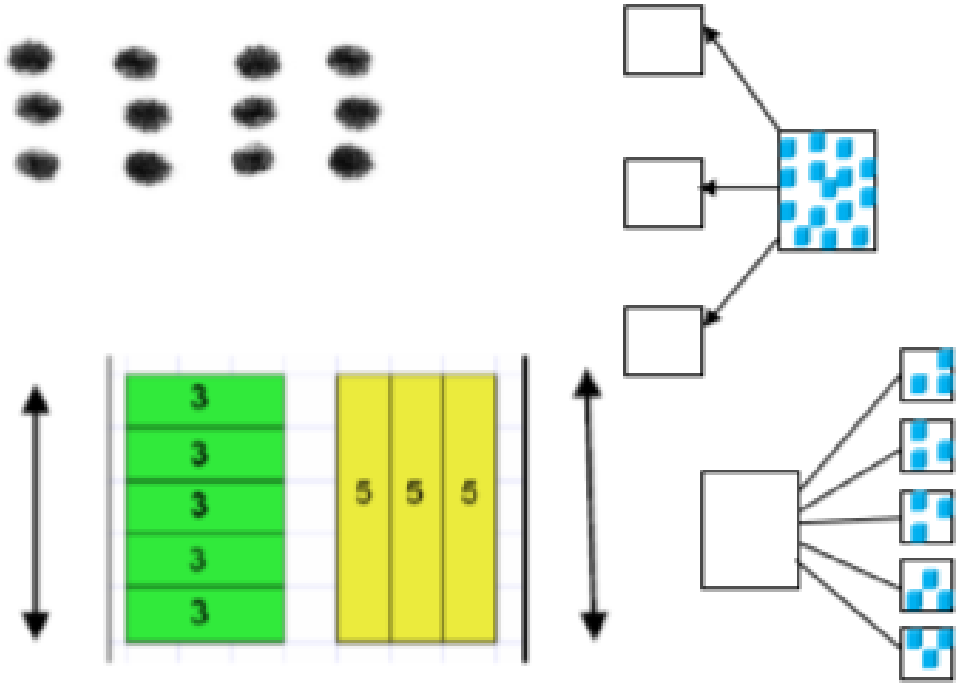
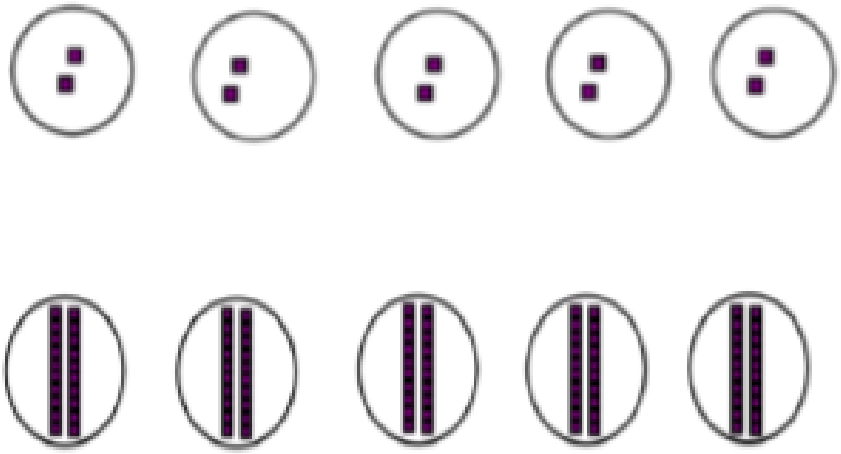
Strategy & guidance	CPA
Add and subtract numbers mentally, including: • **a three-digit number and ones;** • **a three-digit number and tens;** • **a three-digit number and hundreds** *Pupils learn that this is an appropriate strategy when they are able to use known and derived number facts or other mental strategies to complete mental calculations with accuracy.* *To begin with, some pupils will prefer to use this strategy only when there is no need to regroup, using number facts within 10 and derivations. More confident pupils might choose from a range of mental strategies that avoid written algorithms, including (but not exhaustively):* • *known number facts within 20,* • *derived number facts,* • *'Make ten',* • *round and adjust* *See Year 2 guidance for exemplification of these – the use of concrete manipulatives other than Dienes blocks is important in reinforcing the use of these strategies.* *It is important that pupils are given plenty of (scaffolded) practice at choosing their own strategies to complete calculations efficiently and accurately. Explicit links need to be made between familiar number facts and the calculations that they can be useful for and pupils need to be encouraged to aim for efficiency.*	It is important to model the mental strategy using concrete manipulatives in the first instance and pupils should be able to exemplify their own strategies using manipulatives if required, with numbers appropriate to the unit they are working on (3-digit numbers in Units 1 & 4; 4-digit numbers in Unit 13). However, pupils should be encouraged to use known facts to derive answers, rather than relying on counting manipulatives or images. <u>No regrouping</u> 345 + 30 274 - 50 1128 + 300 1312 - 300 326 + 342 856 - 724  I know 4 + 3 = 7, so 4 tens plus 3 tens is equal to 7 tens. 345 + 30 = 375. <u>With some regrouping</u> 416 + 25 232 - 5 383 + 130 455 - 216 611 + 194 130 - 40 1482 + 900 2382 - 500

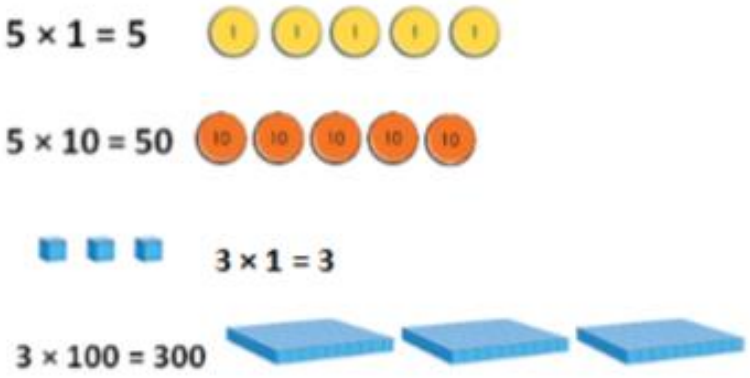
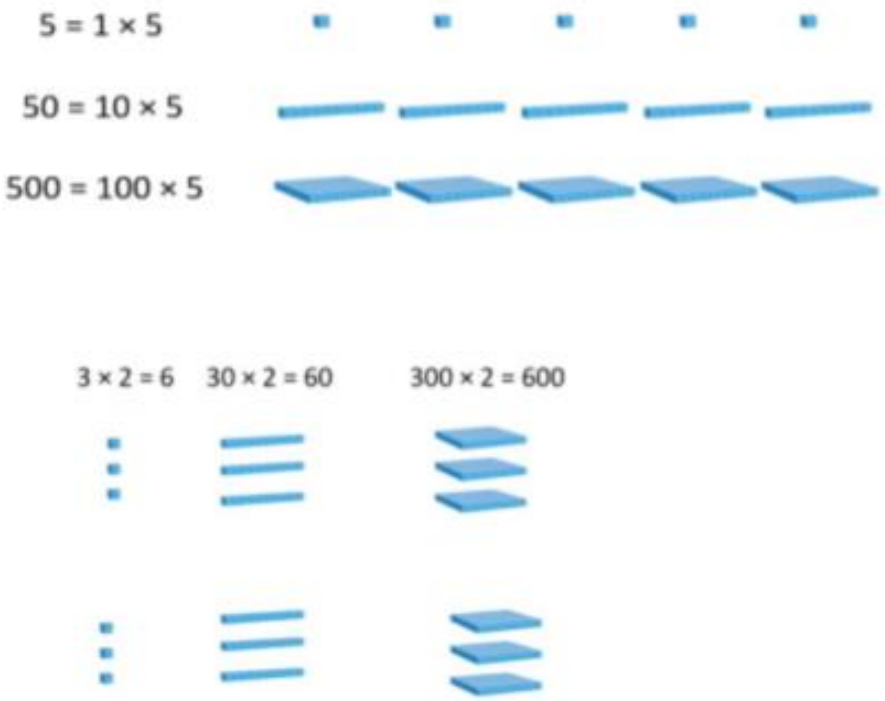
Strategy & guidance	CPA
Written column method for calculations that require regrouping with up to 4-digits *Dienes blocks should be used alongside the pictorial representations during direct teaching and can be used by pupils both for support and challenge. Place value counters can also be introduced at this stage.* *This work revises and reinforces ideas from Key Stage 1, including the focus on place value – see Year 2 exemplification.* *Direct teaching of the columnar method should require at least one element of regrouping, so that pupils are clear about when it is most useful to use it. Asking them 'Can you think of a more efficient method?' will challenge them to apply their number sense / number facts to use efficient mental methods where possible.* *As in Year 2, pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping. In Year 3 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.* *Pupils should be challenged as to whether this is the most efficient method, considering whether mental methods (such as counting on, using known number facts, round and adjust etc.) may be likelier to produce an accurate solution.* *Pupils requiring support might develop their confidence in the written method using numbers that require no regrouping.* *See Unit materials for extra guidance on this strategy.*	As for the mental strategies, pupils should be exposed to concrete manipulatives modelling the written calculations and should be able to represent their written work pictorially or with concrete manipulatives when required. Again, they should be encouraged to calculate with known and derived facts and should not rely on counting images or manipulatives.  5 + 6 = 11 so I will have 11 ones which I regroup for 1 ten and 1 one. <u>Regrouping (including multiple separate instances)</u> 672 + 136 734 – 82 468 + 67 831 - 76 275 + 386 435 – 188 <u>'Regrouping to regroup'</u> 204 – 137 1035 - 851

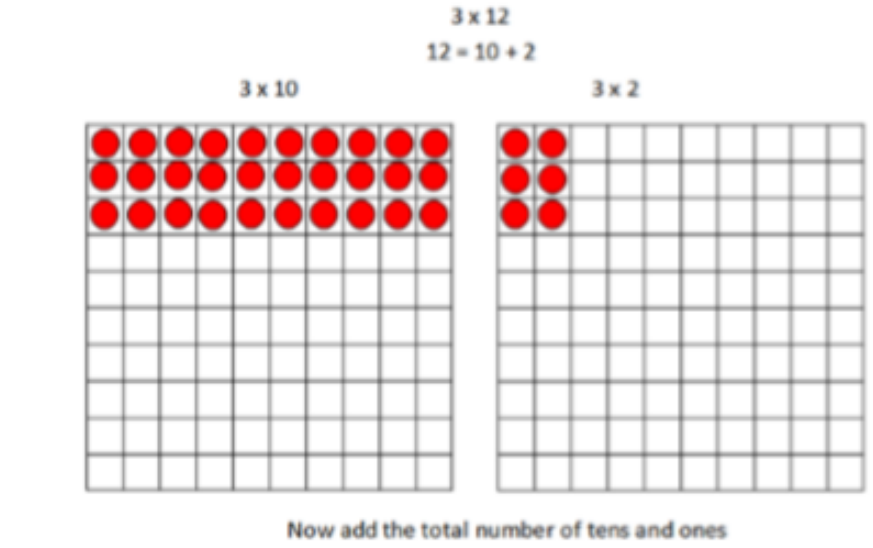
Strategy & guidance	CPA
Find 10, 100 more or less than a given number *As pupils become familiar with numbers up to 1000, place value should be emphasised and comparisons drawn between adding tens, hundreds (and, in the last unit of the Summer term, thousands), including use of concrete manipulatives and appropriate images.* *After initial teaching, this should be incorporated into transition activities and practised regularly.*	142 + 100 = 242  

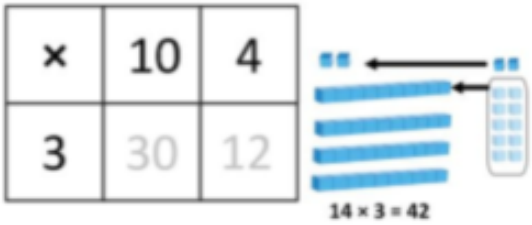
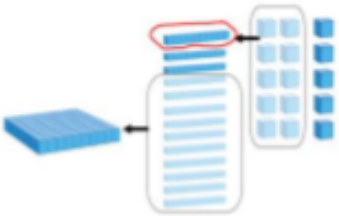
Y3 Multiplication

Strategy & guidance	CPA
Doubling to derive new multiplication facts *Pupils continue to make use of the idea that facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy.* *This builds on the doubling strategy from Year 2.*	3 x 3 = 9  3 x 6 = double 9 = 18 

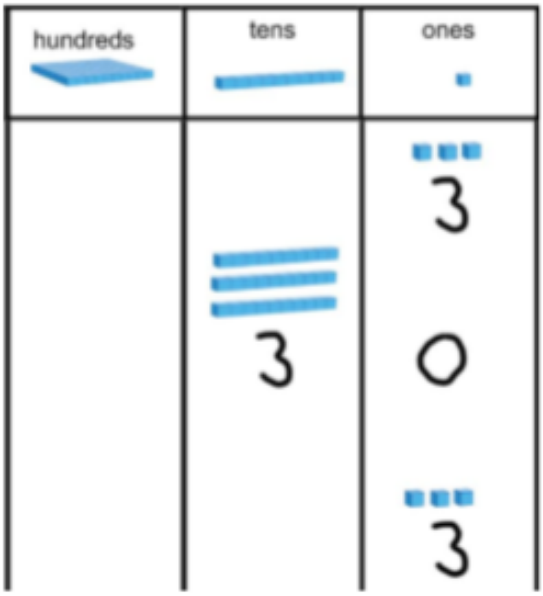
Strategy & guidance	CPA
Skip counting in multiples of 2, 3, 4, 5, 6, 8 and 10 *Rehearsal of previously learnt tables as well as new content for Year 3 should be incorporated into transition activities and practised regularly.*	
Use of part-part-whole model with arrays and bar models to establish commutativity and inverse relationship between multiplication and division *In these contexts pupils are able to identify all the equations in a fact family.*	
Ten times greater *Pupils's work on this must be firmly based on concrete representations – the language of ten times greater must be well modelled and understood to prevent the numerical misconception of 'adding a zero'.*	

Strategy & guidance	CPA
Multiplying by 10 and 100 *Building on the ten times greater work, pupils use appropriate Dienes blocks and place value counters to multiply 2, 3, 4, 5 and 10 by 10, 100 and 1000.*	
Using known facts for multiplying by multiples of 10 and 100 *Pupils' growing understanding of place value, allows them to make use of known facts to derive multiplications using powers of 10.* *It is important to use tables with which they are already familiar (i.e. not 7 or 9 tables in Year 3)*	

Strategy & guidance	CPA
Multiplication of 2-digit numbers with partitioning (no regrouping) *Children should always consider whether partitioning is the best strategy – if it is possible to use strategies such as doubling (some may use doubling twice for ×4), they need to choose the most efficient strategy.* *Children may wish to make jottings, including a full grid as exemplified here – but grid method is not a formal method and its only purpose is to record mental calculations. This supports the development of the necessary mental calculating skills but does not hinder the introduction of formal written methods in Year 4. Concrete manipulatives are essential to develop understanding.*	 | × | 10 | 2 | | --- | --- | --- | | 3 | = | ⁘ | | × | 10 | 2 | | --- | --- | --- | | 3 | 30 | 6 | **3 x 12 = 36**

Strategy & guidance	CPA
Multiplication of 2-digit numbers with partitioning (regrouping) *Using concrete manipulatives and later moving to using images that represent them, supports pupils' early understanding, leading towards formal written methods in Year 4.* *Once again, this is a mental strategy, which they may choose to support with informal jottings, including a full grid, as exemplified here.* *Pupils must be encouraged to make use of their known multiplication facts and their knowledge of place value to calculate, rather than counting manipulatives.*	   

Y3 Division

Strategy & Guidance	CPA
Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down *Pupils use the strategy of 'scaling down', representing numbers with concrete manipulatives and making the value ten times smaller.*	 **3 × 10 = 30** **30 ÷ 10 = 3**
Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using grouping *Pupils divide by 10, 100 and 1000 by making groups of the divisor.*	500 ÷ 100 = ☐ My whole is 500 and the value of the equal parts is 100. How many parts are there?